SIMULATING THE GAMBLER'S RUIN PROBLEM

Lucy Martinez
Mathematics Department

ABOUT ME

- Fourth-year student
- Research: Experimental Mathematics - Game theory

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

PROBABILITY

Concerns events and numerical descriptions of how likely they are to occur

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The probability of an event is a number between zero and one

TYPES OF PROBABILITY

Empirical/Experimental

Estimates probabilities from experience and observation

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Theoretical

Calculates the likeliness of an event happening based on reasoning and mathematics

EXAMPLE 1

Empirical Probability: You toss a coin 8 times and you record the following outcomes

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Outcome	heads	heads	tails	heads	tails	tails	heads	heads

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What is the probability of getting tails? 3/8 = 37.5%

EXAMPLE 2

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$$\frac{\text{number of desired outcomes}}{\text{total number of all outcomes}}$$

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Probability of getting tails? P(tails) = $\frac{1}{2}$ = 50%

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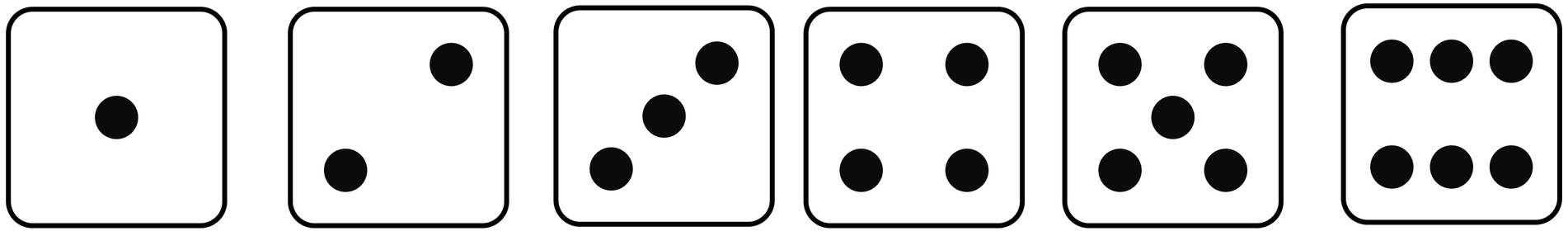
Example: Consider a six-sided die.



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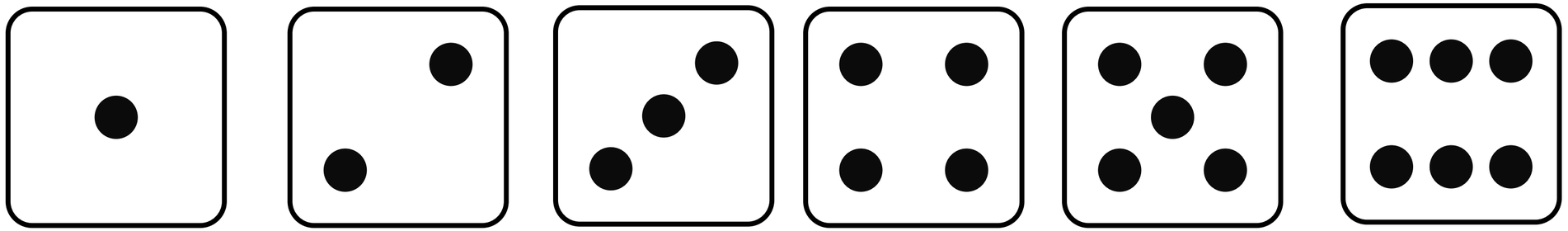

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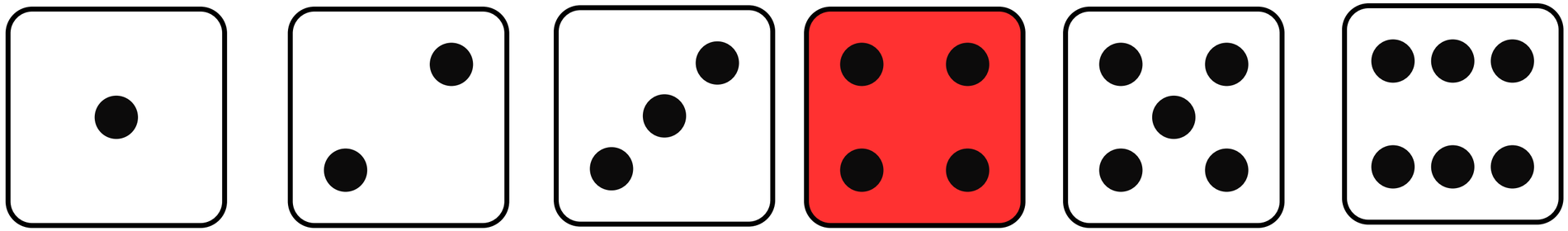
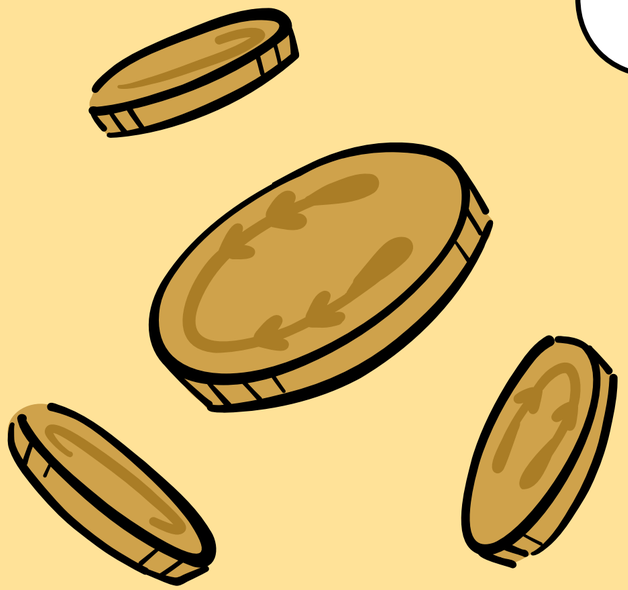
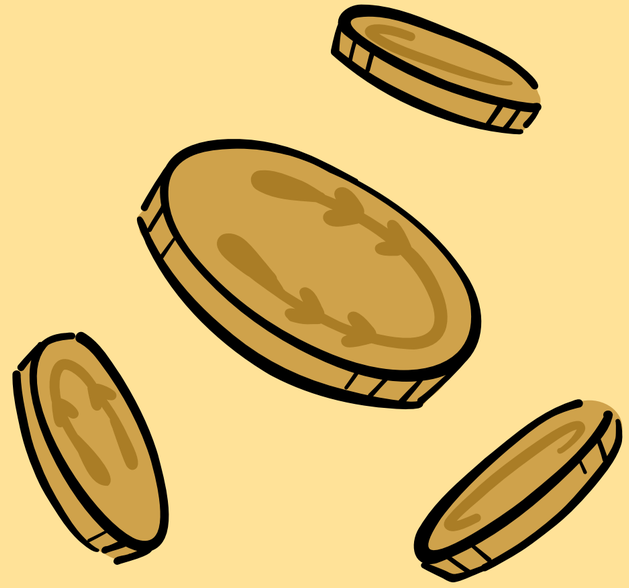


What is the probability of getting a 4?

P(getting a 4) = $\frac{1}{6}$

GAMBLER'S RUIN PROBLEM





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- Hydrology: Sediment transport processes in rivers such as scouring and deposition
- Probability: Random walks
- Other fields...

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The gambler keeps playing until they run out of money or wins a desirable amount. For now, let's say $7

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If the gambler starts with $5, what is the probability of the gambler exiting the game with $7?

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This scenario is known as the gambler's ruin problem, first posed by Blaise Pascal in 1656

EXAMPLE

Gambler starts with $5 and the goal is $7:

Start	$5		$5	
First Round	$6		$4	
Second Round	$5	$7	$3	$5

OBSERVATIONS

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  - Ex: If the gambler is at $4, in the previous step the gambler could have been at $5 or $3
- The gambler either loses $1 or wins $1 with probability $\frac{1}{2}$

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$$P(5) = \frac{1}{2}P(4) + \frac{1}{2}P(6)$$

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  - If the gambler starts with $0 then the gambler ran out of money so the probability of winning $7 is 0
- We know P(7)=1
  - If the gambler starts with $7 then the gambler has reached the goal so the probability of winning $7 is 1

More generally...

We can generalize this scenario by replacing any starting amount by a variable "A" and the goal amount by "N":

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$$P(A) = \frac{1}{2}P(A-1) + \frac{1}{2}P(A+1)$$

Probability Formula

If the gambler starts with $A, what is the probability of the gambler exiting the game with $N?

The formula is...

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$$P(A) = \frac{A}{N}$$

EXAMPLE

If the gambler starts with $5, what is the probability of the gambler exiting the game with $7?

$$P(5) = \frac{5}{7} \approx 71\%$$

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RESEARCH QUESTION

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- The particle moves from A to A+1 or
- The particle moves from A to N-A

What is the probability that the particle reaches the end of the line?

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QUESTIONS?

Website: https://marti310.github.io

Email: lucy.martinez@rutgers.edu